CORRELATION OF FIRST AND SECOND FILTRATION PHASES IN POROUS-FRACTURED RESERVOIR

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We examine the dynamics of bottom-hole pressure recovery in an instantaneously stopped well of a porous-fractured reservoir under the following conditions: 1) the reservoir is infinite; 2) the reservoir is bounded by a circular feed contour which is coaxial with the well.

The entire pressure recovery period in a bounded reservoir breaks down into two phases: first and second. By first phase we mean the time segment $t \geq 0$ in the course of which the pressure recovery takes place nearly (within 1-2%) as if the reservoir were infinite. The remaining time is termed the second phase.

It is shown that for low fracture elastic capacity the first phase may not exist in the porous-fractured reservoir, in contrast with the porous reservoir.

It is known that in the infinite porous [1] and porous-fractured [2] reservoirs the bottom-hole pressure recovery curve for sufficiently large time is approximated asymptotically by some linear function of the logarithm of the time. It does not follow from the definition given above that this logarithmic approximation holds within the limits of the first phase. Therefore, the need arises to introduce the concept of the logarithmicity of the first phase. The logarithmic nature of the first phase forms the basis for the familiar tangential definition of the reservoir geophysical parameters from the bottom-hole pressure recovery curve [1, 2].

Shchelkachev has shown [1] that in a porous reservoir for $r_c \geq 100\, r_w$, the first phase is always logarithmic and the pressure recovery curve for times from the interval $(100\, r_w^2/\varkappa,\ 0.1\, r_c^2/\varkappa)$ admit logarithmic approximation. Here r_c and r_w are the feed contour and well radii, and $\varkappa$ is the piezoconductivity of the reservoir.

The absence of logarithmic behavior follows automatically from the fact that under certain combinations of filtration parameters the first phase may have zero duration in the porous-fractured reservoir. This implies that in this case tangential analysis of the pressure recovery curves is not applicable; so the necessity arises to develop a technique for determining the geological and physical parameters from the relations governing the second filtration phase. Such a technique, which is a development of the ideas of Pollard [3], is presented below.

Ignorance of the fact that logarithmic behavior of the first phase may not exist in the porous-fractured reservoir has been the basis for disagreement between the authors of [4] and [5].

1. Let the function U(r, t) denote the pressure deviation in the fractures of the disturbed reservoir above the steady-state value. In the porous-fractured reservoirs, in which the permeability of the porous blocks is negligibly small, the function U satisfies the equation [2]

$$\varkappa\left(1+\tau\frac{\partial}{\partial t}\right)\nabla^2 U=\left(1+\varepsilon\tau\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}U, \qquad \nabla^2 U=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)$$
$$\varkappa=\frac{k}{\mu(\beta_1+\beta_2)}, \qquad \tau=\frac{\mu}{\beta_2\alpha}, \qquad \varepsilon=\frac{\beta_1}{\beta_1+\beta_2} \tag{1.1}$$

Tyumen'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 6, pp. 134-139, November-December, 1969. Originl article submitted April 23, 1969.

Here r is the radial distance, t is time, ε is the relative elastic capacity, τ is the lag time, k is the fracture permeability, μ is the liquid viscosity, β_1 and β_2 are the elastic capacities of the fractures and porous blocks, α is the mass transfer coefficient of the fractures and blocks, and h is the reservoir thickness.

In studying the dynamics of bottom-hole pressure recovery after "instantaneous" shutdown of a well operating with the flowrate q_0 in the steady state regime, we can without large error assume that the well has zero radius and is a sink (source). Then (1.1) can be solved with the boundary and initial conditions

$$-\frac{2\pi kh}{\mu}\left(r\frac{\partial U}{\partial r}\right)_{r\to 0}=1, \quad U=0 \quad \text{for} \quad r=r_k, \quad U=\frac{\partial U}{\partial t}=0 \quad \text{for} \quad t=0 \tag{1.2}$$

Solving the problem (1.1), (1.2), omitting the details, we find

$$U(r,t)=\frac{\mu q_0}{2\pi kh}\varphi_\varepsilon(\xi,\xi_C,\lambda); \qquad \xi=\frac{r}{\sqrt{\varkappa\tau}}, \quad \xi_C=\frac{r_C}{\sqrt{\varkappa\tau}}, \quad \lambda=\frac{t}{\tau} \tag{1.3}$$

The function φ_ε is defined by the Laplace transformation

$$L\varphi_\varepsilon(\xi,\xi_C,\lambda)=\frac{1}{s}K_0(\xi\zeta)-\frac{1}{s}K_0(\xi_C\zeta)\frac{I_0(\xi\zeta)}{I_0(\xi_C\zeta)}, \quad \zeta=\left(\frac{1+\varepsilon s}{1+s}s\right)^{1/2}, \quad L\varphi_\varepsilon(\xi,\xi_C,\lambda)=\int_0^\infty e^{-s\lambda}\varphi_\varepsilon(\xi,\xi_C,\lambda)\,d\lambda \tag{1.4}$$

Here s is the Laplace transform parameter. In the first filtration phase, when we can assume $r_C=\infty$, the function U is defined by the equality

$$U(r,t)=\frac{\mu q_0}{2\pi kh}\varphi_\varepsilon(\xi,\lambda), \qquad L\varphi_\varepsilon(\varepsilon,\lambda)=\frac{1}{s}K_0(\xi\zeta) \tag{1.5}$$

The expression for $\varphi_\varepsilon(\xi,\lambda)$ is obtained from (1.4) for $\xi_C=\infty$, as follows from the properties of the Macdonald function K_0 and the modified Bessel function I_0.

It is natural to call the functions $\varphi_\varepsilon(\xi,\xi_k,\lambda)$ and $\varphi_\varepsilon(\xi,\lambda)$, respectively, the functions of a linear continuously acting source in finite and infinite porous-fractured reservoirs.

Now we can show that in the porous-fractured reservoir the first phase may have zero duration for some combination of the parameters ξ, τ, $\varkappa$, and r_C.

In fact, we compare $\varphi_0(\xi,\xi_k,\lambda)$ and $\varphi_0(\xi,\lambda)$ for $\xi=0$, when $\zeta=\sqrt{s/(1+s)}$.

The right-hand limits of the functions indicated above as $\lambda\to 0^+$ can be found using operational calculus, for example,

$$\varphi_0(\xi,\xi_C,\lambda)=\lim sF(s), \qquad s\to\infty$$

where F(s) is the right side of (1.4).

Then from (1.4) and (1.5)

$$\varphi_0(\xi,\xi_C,0)=K_0(\xi)-K_0(\xi_C)\frac{I_0(\xi)}{I_0(\xi_C)} \tag{1.6}$$

$$\varphi_0(\xi,0)=K_0(\xi) \tag{1.7}$$

If τ is sufficiently large and, therefore, ε_C is small, (1.6) differs from (1.7) by considerably more than 1-2%, so that the duration of the first phase in such a reservoir equals zero and the second phase occurs immediately.

For $\varepsilon=1$ the functions $\varphi_1(\xi,\xi_k,\lambda)$ and $\varphi_1(\xi,\lambda)$, as follows from their Laplacian transformations (1.4) and (1.5), correspond to the purely porous medium in which the blocks act as grains and the fractures act as pores. However, Shchelkachev [1] has shown that in a porous reservoir the first phase duration is defined by the relation $t\le 0.1\,r_C^2/\varkappa$ and is determined only by the remoteness of the reservoir boundary and the magnitude of the piezoconductivity coefficient.

Consequently, other conditions being the same, the first phase duration is minimal for $\varepsilon=0$ and maximal for $\varepsilon=1$. It will be shown that for $\xi=0$ and a large value of the lag time τ, the first phase duration may be zero.

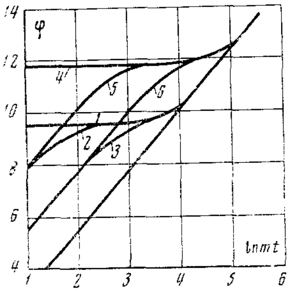

Fig. 1

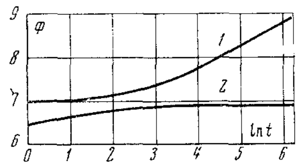

Fig. 2

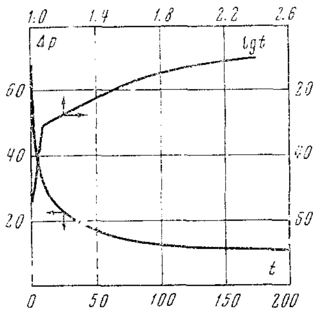

Fig. 3

In order to represent the bottom-hole pressure recovery pattern under conditions of premature influence of the feed contour, we must have available the computational algorithms for the functions $\varphi_\varepsilon(\xi, \xi_c\ \lambda)$ and $\varphi_\varepsilon(\xi, \lambda)$.

2. An explicit expression for $\varphi_\varepsilon(\xi, \xi_c\ \lambda)$ can be found from the Riemann–Mellin inversion formula with use of the residue theorem, if we make use of the fact that the Laplace transform F(s) of this function is meromorphic in the left half-plane of the complex variable s with poles at the points $s=0$, $s=-a_n$ and $s=-b_n$ $(n=1, 2, 3, \ldots)$.

The quantities $-a_n$ and $-b_n$ will be the roots of the quadratic equation

$$(\varepsilon_c \zeta)^2 = -\eta_n$$

whose solution yields

$$2\varepsilon a_n = 1 + \eta_n\theta - d_n, \qquad 2\varepsilon b_n = 1 + \eta_n\theta + d_n, \qquad d_n = [(1+\eta_n\theta)^2 - 4\varepsilon\eta_n\theta]^{1/2}, \qquad \theta = \xi_c^{-2} \tag{2.1}$$

Thus we have

$$\varphi_\varepsilon(\xi, \xi_c, \lambda) = -\ln\rho - \sum_{n=1}^{\infty} C_n \left[\frac{1-\varepsilon a_n}{d_n} e^{-a_n\lambda} + \frac{\varepsilon b_n - 1}{d_n} e^{-b_n\lambda}\right] J_0(\rho\sqrt{\eta_n})$$
$$C_n = 2/\eta_n J_1^2(\sqrt{\eta_n}), \qquad \rho = \xi/\xi_c \tag{2.2}$$

Here J_0 and J_1 are Bessel functions of zero and first order of the first kind, and $\sqrt{\eta_n}$ are roots of the equation $J_0(\sqrt{\eta_n}) = 0$.

For $\varepsilon=0$ (2.2) coincides with the Avakyan solution [6]; for $\varepsilon=1$ it coincides with the Muscat solution of the boundary value problem (1.2) for the piezoconductivity equation [1].

The same Riemann–Melling inversion formula can be used to obtain an explicit expression for the function $\varphi_\varepsilon(\xi, \lambda)$. This expression will be quite complex and not suitable for computational purposes.

The following approximate formula yields adequate accuracy for the study of pressure recovery wells:

$$\varphi_\varepsilon(\xi, \lambda) \approx \varphi_-(\xi, \lambda) - \frac{1}{2}\left[\ln\frac{4\lambda}{\gamma\xi^2} - \mathrm{Ei}(-\lambda) + \mathrm{Ei}\left(-\frac{\lambda}{\varepsilon}\right)\right] \tag{2.3}$$

Here $\mathrm{Ei}(-\lambda)$ is the exponential integral function.

This will be the lower approximation and can be obtained as follows. We use the equality

$$-\frac{1}{2}\mathrm{Ei}\left(-\frac{\xi^2}{4\lambda}\right) = \frac{1}{2}\ln\frac{4\lambda}{\gamma\xi^2} + \frac{1}{2}\int_0^\infty F(\xi, \mu)[1 - h(\lambda-\mu)]\,d\mu$$
$$F(\xi, \mu) = \mu^{-1}[1 - \exp(-\xi^2/4\mu)]$$

Here $h(\lambda)$ is the Heaviside function, and $\gamma = 1.7810\ldots$ is the Euler constant.

In Laplace transforms the last equality takes the form

$$K_0(\xi\sqrt{s})=\ln\frac{2}{\gamma\xi\sqrt{s}}+\frac{1}{2}\int_0^\infty F(\xi,\mu)\left[1-e^{-s\mu}\right]d\mu \tag{2.4}$$

Setting here ζ in place of $\sqrt{s}$, we obtain

$$\frac{1}{s}K_0(\xi\zeta)=\frac{1}{s}\ln\frac{2}{\gamma\xi\zeta}+\frac{1}{2s}\int_0^\infty F(\xi,\mu)\left[1-\exp-\zeta^2\mu\right]d\mu \tag{2.5}$$

Change in this equality to the originals yields with account for (1.5) and the sense of the quantity ζ:

$$\varphi_\varepsilon(\xi,\lambda)=\varphi_-(\xi,\lambda)+\frac{1}{2}\int_0^\infty F(\xi,\mu)\left[1-Q(\lambda,\mu)\right]d\mu$$

The function $Q(\lambda, \mu)$ is defined by the operational transform

$$LQ(\lambda,\mu)=s^{-1}\exp(-\zeta^2\mu),$$

from the expansion of which into a series in $[s(1+s)^n \exp \varepsilon s\mu]^{-1}$, and then converting to the originals, we obtain

$$1>Q(\lambda,\mu)>e^{-\mu}h(\lambda-\varepsilon\mu)$$

The first of these inequalities together with (2.5) yields

$$\varphi_\varepsilon(\xi,\lambda)>\varphi_-(\xi,\lambda)$$

i.e., (2.3) actually will be the lower approximation. The second inequality makes possible the upper evaluation of the error of (2.3). The result is

$$\frac{1}{2}\int_0^\infty F(\xi,\mu)\left[1-Q(\lambda,\mu)\right]d\mu<\frac{1}{2}\int_0^{\lambda/\varepsilon} F(\xi,\mu)(1-e^{-\mu})\,d\mu+\frac{1}{2}\int_{\lambda/\varepsilon}^\infty F(\xi,\mu)\,d\mu$$

We have

$$R_1=\frac{1}{2}\int_0^\infty F(\xi,\mu)(1-e^{-\mu})\,d\mu=K_0(\xi)-\ln\frac{2}{\gamma\xi}$$

which is obtained from (2.4) by replacing s by 1. Further

$$\frac{1}{2}\int_{\lambda/\varepsilon}^\infty F(\xi,\mu)\,d\mu<\frac{\varepsilon\xi^2}{4\lambda}=R_2$$

Thus, the error of the approximation (2.3) is no greater than

$$(R_1+R_2)\,/\,\varphi_-(\xi,\lambda)$$

This implies that the error of (2.3):

a) is no more than 0.7% for $\xi\le 0.1$ and $\xi^2/4\lambda\le 0.01$ for $0<\varepsilon<1$;

b) is no more than 0.35% for $\xi\le 0.1$ for $\xi=0$;

c) is no more than 0.5% for $\xi^2/4\lambda\le 0.01$ for $\varepsilon=1$.

Thus, the approximation (2.3) is quite acceptable for studying unsteady processes in the immediate vicinity of a source and even more acceptable for analysis of bottom-hole pressure recovery.

In accordance with these results (Eqs. (1.5) and (2.3) for $r=r_w$), the pressure increase $\Delta p_w(t)$ at the bottom of the instantaneously stopped well of a porous-fractured reservoir in the first filtration phase is expressed by the formula

$$U(r_w\,t)=\Delta p_w(t)=\frac{\mu q_0}{4\pi kh}\left[\ln\frac{4}{\gamma}mt-\mathrm{Ei}\left(-\frac{t}{\tau}\right)+\mathrm{Ei}\left(-\frac{t}{\varepsilon\tau}\right)\right]\quad\left(m=\frac{\varkappa}{r^2}\right) \tag{2.6}$$

Formula (2.6) differs from the corresponding formula for the porous reservoir

$$\Delta p_w(t) = \frac{\mu q_0}{4\pi kh} \ln \frac{4}{\gamma} mt \tag{2.7}$$

in the last two terms, which are negligibly small for large values of the time t. Therefore, in the semi-logarithmic anamorphism, the curve (2.6) will have (2.7) as its rectilinear asymptote, as shown in Fig. 1, where the ordinate axis is the quantity

$$\psi = \frac{4\pi kh}{\mu} \frac{\Delta p}{q_0}$$

and the curves 1, 2, ..., 6 are defined by the following values of mt and ε:

Curves	1	2	3	4	5	6
$m\tau =$	10^4	10^4	10^4	10^5	10^5	10^5
$\varepsilon =$	0	0.01	0.1	0	0.01	0.1

If the first phase is logarithmic, the slope of the logarithmic asymptote is used to determine in the usual way the reservoir hydraulic conductivity kh/μ. Otherwise the influence of the feed contour distorts markedly the pressure recovery process, as is seen in Fig. 2.

The upper curve in this figure corresponds to an infinite reservoir; the lower corresponds to a reservoir bounded by a circular feed contour. The following values of the parameers were used in plotting the curves $\tau = 20$ h, $\varepsilon = 0$, $r_w = 0.8$ 485 cm, $r_c = 84.85$ m, i.e., $\xi_w = 0.001$, $\xi_c = 1$.

The somewhat artificial values given to the well and feed contour do not alter the physical essence of the phenomena; however, they do permit considerable simplification of the calculations. The values of the other parameters are taken equal to the average values for the Kyurovdag field in the Azerbaidzhan SSR.

We see from Fig. 2 that the bottom-hole pressure recovery curve under the influence of the feed contour may not have a logarithmic asymptote, and therefore the tangent method is not applicable in this situation.

3. In the present section we describe a method for determining the geological and physical parameters of a reservoir based on formulas (1.3) and (2.2), which describe the second filtration phase under the assumption that the reservoir boundary is a circular feed contour. The need to use this method arises when the first pressure recovery phase is not logarithmic. In this case, the influence of the oil reservoir boundaries will be very significant and cannot be ignored. Following Charnyi [7], who first proposed a calculation of this type for a porous reservoir, we assume that the real reservoir admits approximation by a circle of radius r_c with constant contour pressure p_c.

Then the pressure recovery in the later filtration phase period will be described by the formula

$$\Delta p(t) = Ae^{-\alpha t} + Be^{-\beta t} \quad (\Delta p(t) = p(t) - p_k) \tag{3.1}$$

Here $\Delta p(t)$ is the deviation of the instantaneous bottom-hole pressure p(t) from the reservoir pressure p_c, equal to the pressure on the feed contour $r = r_c$

$$A = \frac{\mu q_0}{2\pi kh} C_1 \frac{1 - \varepsilon\tau\alpha}{d_1}, \qquad B = A\frac{\varepsilon\tau\beta - 1}{1 - \varepsilon\tau\alpha}, \qquad \alpha = \frac{a_1}{\tau}, \qquad \beta = \frac{b_1}{\tau}, \qquad C_1 = \frac{2}{\eta_1 J_1^2(\sqrt{\eta_1})} = 1.283 \tag{3.2}$$

Formula (3.1) is obtained from (1.3) and (2.3) by neglecting all terms of the finite sum other than the first two. For sufficiently large values of the time these two terms are considerably larger than the remaining terms.

If, beginning at some time, the actual curve is described sufficiently well by the binomial (3.1), the coefficients A, B, α, and β canbe determined by graphical constructions [3] or analytically using the Lanczos technique [8].

The definitions of these quantities lead to the equalities

$$\frac{B}{A} = \frac{\varepsilon\tau\beta - 1}{1 - \varepsilon\tau\alpha}, \qquad \alpha + \beta = \frac{1 + \eta_1\theta}{\varepsilon\tau}, \qquad \alpha\beta = \frac{\varepsilon\eta_1\theta}{(\varepsilon\tau)^2}$$

Solving these equations for the unknowns, we have

$$\eta_1\theta = (\alpha+\beta)\delta - 1,\ \varepsilon\eta_1\theta = \alpha\beta\delta^2,\ \tau\varepsilon = \delta,\ \delta = (A+B)/(A\alpha + B\beta) \tag{3.3}$$

The values of θ, ε, and τ are easily found from (3.3) in the indicated sequence.

Since $d_1 = (\beta - \alpha)\varepsilon\tau$, it follows from (3.2) that

$$\frac{kh}{\mu} = 0.2\,\frac{1-\varepsilon\tau\varkappa}{(\beta-\alpha)\varepsilon\tau}\,\frac{q_0}{A} \tag{3.4}$$

From the known values of θ and τ we find

$$\varkappa / r_k^2 = \theta / \tau$$

This technique is illustrated using as an example Well 24 of the Strel'nenskii field in the Kuibyshev Region, taken from [9]. Prior to shutdown this well operated with a constant flowrate $q_0 = 430$ m³/day = 1977 cm³/sec. This was a free-flowing well so that the shutdown could be considered instantaneous. The effective reservoir thickness was 5 m. The reservoir is porous-fractured.

The instantaneous bottom-hole pressure deviation from the reservoir pressure is shown in Fig. 3 as a function of time and its decimal logarithm. We see from this figure that the pressure variation curve in the semilogarithmic version is the initial portion of the curves shown in Fig. 1, prior to their approach to the asymptote. The absence of the logarithmic asymptote in this case is most probably explained by the influence of the feed contour.

The Lanczos technique was used to find the following values of the coefficients: A = 12.3 atm, B = 23 atm, $\alpha = 9.2\cdot10^{-6}$ sec^{-1}, $\beta = 5.09\cdot10^{-4}$ sec^{-1}. The correctness of these values is confirmed by comparison of the actual values of Δp_1 (atm) with the values of Δp_2 (atm) calculated using (3.1) for the indicated values of the coefficients

$t =$	0	12.5	25	50	75	100
$\Delta p_1 =$	67	37	27.2	16.8	14.2	12.6
$\Delta p_2 =$	35.7	27.92	22.88	16.95	14.14	12.71
$t =$	125	150	175	200	225	250
$\Delta p_1 =$	11.9	11.2	11.1	11.0	10.7	10.6
$\Delta p_2 =$	11.97	11.57	11.27	11.07	10.77	10.61

The agreement between the actual and calculated values for $t \geq 50$ min is quite satisfactory. Therefore, in this case the second filtration phase begins at approximately this time.

From (3.3) we find $\theta = 0.344$, $\varepsilon = 0.085$, $\tau = 6.6\cdot10^{-4}$ sec.

From (3.4) and (3.5) we have

$$kh/\mu = 29.6 \text{ Darcy}\cdot\text{cm/cp},\ r_k^2 = 2\cdot10^5\varkappa \text{ sec.}$$

In conclusion, we note that a formula of the form (3.1) was first proposed for analysis of well pressure recovery curves in a porous-fractured collector by Pollard [3] who, however, was not able to justify his recommendation rigorously, with the result that he did not clarify the physical meaning of the coefficients A, B, α, and β.

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